H-ARAIM Exclusion: Requirements and Performance

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September 16, 2016
Background

- Two key developments in future GNSS:
  - **Dual Frequency Signal**: reduce measurement error
  - **Multi-Constellation**: provide more measurement redundancy

are expected to bring significant navigation performance improvement in civil aviation using RAIM method [1].

- RAIM employs redundant measurements to achieve self-contained fault detection and exclusion (FDE) [2].

- Advanced RAIM (ARAIM) will serve for applications with more stringent navigation requirements [3].

[1] Phase II of the GNSS Evolutionary Architecture Study, February 2010


Introduction

- Horizontal ARAIM (H-ARAIM) is currently of primary interest [4].
  - H-ARAIM aims at providing horizontal navigation service for the aircraft during en-route flight, terminal, non-precision approach (NPA), etc.

➤ Detection function:
  - Ensure Integrity

➤ Exclusion function:
  - Maintain Continuity

Case 1: Only Detection Function
- Stop Using GNSS
  - Fault Detected
  - Go

Case 2: Detection and Exclusion
- Continue Using GNSS
  - Fault Detected, Excluded
  - Go

Outline

• H-ARAIM Exclusion and Continuity:
  – Interpret H-ARAIM continuity requirements, show that exclusion is required.
  – Assess the impact of different sources on H-ARAIM continuity, and quantify the overall continuity risk.

• Describe H-ARAIM FDE algorithm, quantify predictive FDE integrity risk.
  – Introduce a computationally efficient upper bound on integrity risk, analyze its tightness.

• Evaluate the overall predicted FDE availability.
  – Show the availability performance for H-ARAIM targeted service.
  – Address the impact of unscheduled satellite outages on continuity.
Navigation Requirements

- For H-ARAIM service, both misleading information and loss of continuity (LOC) are specified as major failure conditions [5].

<table>
<thead>
<tr>
<th>Horizontal Alert Limit (HAL)</th>
<th>Integrity Risk $I_{REQ}$</th>
<th>Continuity Risk $C_{REQ}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RNP 0.1</td>
<td>0.1nm (185m)</td>
<td>$10^{-7}$/hour</td>
</tr>
<tr>
<td>RNP 0.3</td>
<td>0.3nm (556m)</td>
<td>$10^{-8}$/hour to $10^{-4}$/hour</td>
</tr>
</tbody>
</table>

- To declare the service being available, both $I_{REQ}$ and $C_{REQ}$ need to be met.
  - RNP 0.1/0.3 are used as examples to illustrate H-ARAIM performance.

Need of H-ARAIM Exclusion

- The range of the continuity risk accounts for the number of aircraft using the same service.
  - “Intermediate values of continuity (e.g. $1 - 1 \times 10^{-6}$ per hour) are considered to be appropriate for areas of high traffic density and complexity where there is a high degree of reliance on the navigation system but in which mitigation for navigation system failures is possible.” [ICAO Annex 10]

- In this work, we use: $C_{REQ} = 10^{-6} / \text{hour}$ [7].
  - Consider a typical example case for H-ARAIM: two constellations, 16 satellites in view, $R_{sat} = 10^{-5}/\text{hour}$ and $R_{const} = 10^{-4}/\text{hour}$ [8].
  - Without exclusion, the probability of LOC due to detection is:
    
    \[
    10^{-5} / \text{hour} / \text{SV} \cdot 16 \text{ SVs} + 10^{-4} / \text{hour} = 2.6 \cdot 10^{-4} / \text{hour} \gg C_{REQ}
    \]
  - Therefore, H-ARAIM exclusion is required for navigation continuity.

With exclusion implemented, H-ARAIM LOC can result from any of the following:

- Not excluded false alarm (NEFA), not excluded fault detection (NEFD), unscheduled satellite outage (USO), radio frequency interference (RFI), and ionospheric scintillation (IOSC).

The probability of H-ARAIM LOC is:

\[ P_{LOC} = P_{NEFA} + P_{NEFD} + P_{USO} + P_{RFI} + P_{IOSC} \]

(1)
H-ARAIM LOC Tree

Total SIS Loss of Continuity

\[ P_{\text{LOC}} < C_{\text{REQ}} (10^{-6} / \text{hr}) \]

- \(< 10^{-7} / \text{hr}\)
  - RFI, IOSC
  - \(P_{\text{RFI}} + P_{\text{IOSC}}\)

- \(< 10^{-7} / \text{hr}\)
  - Unscheduled critical satellite outages
  - \(P_{\text{USO}}\)

- \(< 4 \times 10^{-7} / \text{hr}\)
  - Not Excluded Fault Detection
  - \(P_{\text{NEFD}}\)

- \(< 4 \times 10^{-7} / \text{hr}\)
  - Not Excluded False Alarm
  - \(P_{\text{NEFA}}\)

Number of Critical SVs

- \(P_{\text{OUT}}\)

Unexpected SV(s) loss

\[ 2 \times 10^{-4} / \text{hr} / \text{SV} \] (more later)

No exclusion

Fault detected

\[ 10^{-5} / \text{hr} / \text{SV} \]

SIS fault occurs

Fault-free (FF) state

\[ 1 - 10^{-5} / \text{hr} / \text{SV} \]
\( C_{REQ} \) Allocation

- Not excluded false alarm (NEFA):

\[
P_{\text{NEFA}} < P(D \mid FF) \ P_{\text{FF}} < 4 \times 10^{-7} / \text{hr} \tag{2}
\]

  - The probability of fault free (FF) detection could be limited by setting the detection threshold.

- Not excluded fault detection (NEFD):

\[
P_{\text{NEFD}} < P(NE \mid F) \ P_{F} < 4 \times 10^{-7} / \text{hr} \tag{3}
\]

  - The probability of no exclusion (NE) when faults occur could be limited by setting the exclusion threshold.

- RFI + IOSC:

  - These two impacts are not quantified, and we assume \( P_{RFI} + P_{IOSC} < 10^{-7} / \text{hr} \) is always true in this work.
The impact of USO on H-ARAIM continuity is [9]:

\[ P_{USO} = n_c \cdot P_{OUT} < 10^{-7} / \text{hr} \]  

- \( P_{OUT} \) is the occurrence rate of USO: \( 2 \times 10^{-4} / \text{hr/SV} \) [10].
- \( n_c \) is the number of critical satellites. A critical satellite is the one whose loss leads to LOC during flight.
- Eqn. (4) is equivalent to: \( n_c < 5 \times 10^{-4} \) SV, which indicates no critical satellite is allowed to exist for H-ARAIM applications.

Determine a critical satellite:

- For a geometry where \( P_{HMI} < I_{REQ} \), if removing a satellite results in \( P_{HMI} > I_{REQ} \), then the removed satellite is regarded as a critical satellite.
- Therefore, \( n_c \) depends on the method of evaluating \( P_{HMI} \) (or PL).

• This algorithm is based on solution separation (SS) method.
  – Motivated from improving H-ARAIM continuity, this algorithm could be extended to other applications.

• The flow diagram described the FDE procedure in real time.

FDE Flow Diagram

- Measurements (may be faulted)
- All-in-View Detection
  - Yes
  - No
- Find Subset(s) to Exclude
  - Yes
  - No
- Evaluate $P_{HMI}$ (or PL)
  - Yes
  - No
- $P_{HMI} < I_{REQ}$
  - Yes
  - No
- Continue
- LOC

Exclusion Threshold
Real Time FDE Algorithm

- **Summary of implementing this algorithm in real time:**
  - **Step 1:** Using all in view satellites, if there is no fault detection ($D_0$), go to step 4; if a fault detection ($D_0$) occurs, go to step 2.
  
  - **Step 2:** Array the normalized detection statistics in a magnitude descending order. This order is called “exclusion option order”.
    
    - Example:
      
      Statistics: $|q_3|, |q_7|, |q_1|, |q_5|, \ldots |q_h|, |q_2|$
      
      Order: 1st, 2nd, 3rd, 4th, \ldots
      
    - **Step 3:** Follow the order made in step 2, employ a *second layer detection test* for each option. The first option that passes this test is $E_j$.
    
  - **Step 4:** Evaluate the integrity risk (or PL) using the present satellites.
To predict the FDE integrity risk, all exclusion options must be accounted for:

No Fault Detection ($\overline{D_0}$), and user is in hazardous state ($HI_0$)

Fault is detected ($D_0$) and $j$ is excluded ($E_j$), and user is still in hazardous state ($HI_j$)

$$P_{HMI} = P(HI_0, \overline{D_0}) + \sum_{j=1}^{h} P(HI_j, E_j, D_0)$$  \hspace{1cm} (6)

According to the algorithm, two conditions will result in $j$ being excluded:

- No second layer detection after excluding $j$: $\overline{D_j}$
- $j$ corresponds to the maximum statistic among the subsets that pass the second layer detection test: $\text{MAX}_j$
Multiple Fault Hypothesis

- Account for all fault hypothesis, Eqn. (6) becomes:

\[ P_{HMI} \leq \sum_{i=0}^{h} \max_{f_i} \left\{ P(HI_0, D_0 | H_i, f_i) + \sum_{j=1}^{h} P(HI_j, D_j, \text{MAX}_j, D_0 | H_i, f_i) \right\} P_{Hi} + P_{NM} \tag{7} \]

  - \( P_{NM} \): probability of rarely fault occurring (Not Monitored).
  - \( Hi \): fault mode from \( i = 0 \ldots h \).
  - \( f_i \): fault vector corresponds to fault mode \( i \).

- Employ an example to illustrate in parity space:

  - Measurement Model: \( z = Hx + v + f \) \tag{8}
    - where, \( H = [1 \ 1 \ 1]^T \) and \( v \sim N(0_{3x1}, I_3) \)
  - Only consider single fault mode. Assuming the fault is on \( i = 1 \).
The conditional FDE integrity risk for $H_1$ is:

$$P_{HMI,H_1} = \max_{f_1} \left( P(HI_0, \overline{D}_0 \mid H_i, f_1) + P(HI_1, \overline{D}_1, \text{MAX}_1, D_0 \mid H_1, f_1) + \sum_{j=2}^{3} P(HI_j, \overline{D}_j, \text{MAX}_j, D_0 \mid H_1, f_1) \right) P_{H_1}$$

(9)
Practical Approach

- An upper bound of the FDE integrity risk is used [11].

\[
P_{HMI} \leq \sum_{i=0}^{h} \max_{f_i} \left( P(HI_0, D_0 | H_i, f_i) + \sum_{j=1}^{h} P(HI_j, D_j, MAX_j, D_0 | H_i, f_i) \right) P_{Hi} + P_{NM} \quad (7)
\]

\[
\leq \sum_{i=0}^{h} \max_{f_{i,0}} P(HI_0, D_0 | H_i, f_{i,0}) P_{Hi} + \sum_{i=0}^{h} \sum_{j=1}^{h} \max_{f_{i,j}} P(HI_j, D_j | H_i, f_{i,j}) P_{Hi} + P_{NM} \quad (10)
\]

- Two conservative steps from Eqn. (7) to (10):
  
  - The knowledge of \( MAX_j \) and \( D_0 \) are not used.
  - The risks in Eqn. (10) are maximized individually for same hypothesis.

- However, using Eqn. (10) could potentially cause a loose bound. (next slides).

• The expression of the bound in parity space is:

\[ D_0, E_1 > D_0, E_2 \]

- (c) and (d) may cause loose bound since the red region overlaps with the actual fault mode line.
- The tightness of this bound could be investigated by comparing the bound with numerical results.
Tightness of the Bound

- To investigate the tightness of the bound, Monte-Carlo simulation is employed for this example.
  - Run $10^7$ trials, standard deviation $\sigma = 1m$, prior probability $10^{-3}$ and false alarm requirement is set to be $10^{-6}$.
  - The numbers in the table are predictive FDE integrity risk corresponding their requirements. *The exclusion requirement in case 2 is more stringent than case 1.* (more results in paper)

Table 2. Comparison of the Numerical Results and Bound

<table>
<thead>
<tr>
<th></th>
<th>AL = 4m</th>
<th></th>
<th>AL = 5m</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Numerical</td>
<td>Bound</td>
<td>Numerical</td>
<td>Bound</td>
</tr>
<tr>
<td>Case 1</td>
<td>2.43 x $10^{-6}$</td>
<td>7.37 x $10^{-5}$</td>
<td>2.92 x $10^{-8}$</td>
<td>1.91 x $10^{-6}$</td>
</tr>
<tr>
<td>Case 2</td>
<td>4.03 x $10^{-6}$</td>
<td>7.62 x $10^{-4}$</td>
<td>7.45 x $10^{-7}$</td>
<td>6.67 x $10^{-5}$</td>
</tr>
</tbody>
</table>

- Tighten the FDE integrity bound is not the focus of this work, and it will be considered in future work.
H-ARAIM Simulation

- In this work, integrity risk bound is used to analyze H-ARAIM FDE performance:
  - Computationally efficient.
  - Guarantee safety.

- Baseline simulation conditions:
  - Nominal error model
  - Dual-frequency, baseline GPS/Galileo constellation

<table>
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<tr>
<th>Table 3. Simulation Parameters</th>
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<tbody>
<tr>
<td>Integrity Risk $I_{REQ}$</td>
</tr>
<tr>
<td>$P_{NEFA, REQ}$</td>
</tr>
<tr>
<td>$P_{NEFD, REQ}$</td>
</tr>
<tr>
<td>$P_{sat}$</td>
</tr>
<tr>
<td>$P_{const}$</td>
</tr>
<tr>
<td>$\sigma_{URA}$</td>
</tr>
<tr>
<td>$b_{nom}$</td>
</tr>
<tr>
<td>Mask Angle</td>
</tr>
<tr>
<td>Coverage Range</td>
</tr>
</tbody>
</table>
The results show the predicted H-ARAIM FDE availability performance of $P_{HMI} < I_{REQ}$.

In comparison with detection only, continuity is improved by implementing exclusion.
• Recall: $C_{req}$ could be met only if $n_c = 0$.

- At many locations, $n_c = 0$. At locations where $n_c \neq 0$, the occurrence of USO on critical satellites could impact H-ARAIM continuity.

- However, an upper bound is used to achieve this analysis. This bound may reduce the robustness to satellite geometry, declare a satellite to be ‘critical’ when it actually is not.
Conclusion

- Due to the stringent continuity requirement, fault exclusion is needed for H-ARAIM applications.

- By implementing the FDE algorithm described in this presentation:
  - H-ARAIM continuity could be significantly improved.
  - *High availability performance could be achieved for H-ARAIM.*

- From the critical satellite analysis:
  - The occurrence of USO have a noticeable impact on H-ARAIM continuity.
  - This impact may be mitigated by tighten the FDE integrity bound, and we are investigating it.
Acknowledgement

We would like to thank the Federal Aviation Administration for sponsoring this work.
• Results show that there are more critical satellites in the mid-latitude region.

• Since the average critical satellite number is a reflection of the satellite geometry, horizontal dilution of precision (HDOP) could be used to illustrate this trend.
• To evaluate the critical satellite number $n_c$:

(1) At a location and a time epoch, evaluate $P_{HMI}$ (or PL). If $P_{HMI} < I_{REQ}$, then go to step 2, otherwise, $n_c = 0$.

(2) Remove one satellite and reevaluate $P_{HMI}$. If $P_{HMI} > I_{REQ}$, then the removed satellite is regarded as a critical satellite. Otherwise, it is not a critical satellite.

(3) Repeat step 2 for all the in view satellites, record all the critical satellites.

(4) Sum up the number of critical satellites in step 3, the number is $n_c$ for that location and time epoch.