High-Integrity TLE Error Models for MEO and GEO Satellites

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This paper describes a statistical positioning error analysis for two-line element (TLE) data of a set of medium earth orbiting (MEO) and geostationary earth orbiting (GEO) satellites over the past 20 years. The Joint Space Operations Center (JSpOC) provides spacecraft ephemerides in the form of TLEs, which represent the most comprehensive catalogue of space objects readily accessible to the public. Its utility as an initial estimate tool for space operations has made its limitations a topic of extensive study for many years. In this paper, we compare TLE data with precise ephemerides for tens of thousands of TLE sets, for representative space objects including Global Positioning System (GPS) and Wide Area Augmentation System (WAAS) satellites. Based on this data, we derive high-integrity error models using overbounding theory to determine probabilistic bounds on the radial, along-track, and cross-track positioning errors.

I. Introduction

This paper describes and implements a methodology for high-integrity TLE error modeling for two example orbital regimes: medium earth orbit (MEO) and geostationary earth orbit (GEO). The development of this probabilistic model leverages prior work in aviation navigation safety, where non-Gaussian error distributions are robustly accounted for using overbounding theory [1, 2].

Space is getting increasingly congested. As of January 2018, there are about 23,000 space objects regularly tracked by the U.S. Space Surveillance Network (SSN) and maintained in their catalogue. There is an estimated 29,000 objects larger than ten centimeters, 750,000 sub-decimeter-size objects that cannot be monitored, and another 166 million sub-centimeter-size objects [3]. This causes a significant risk to spacecraft operations considering that a collision with a five-centimeter-size object at low earth orbit (LEO) would release an amount of energy comparable to that of being hit by a bus [4]. Research on Space Situational Awareness (SSA) was strongly energized by the 2007 intentional destruction by China of its Fengyun-1C satellite, and by the 2009 collision of an operating Iridium telecommunication satellite with a decommissioned Cosmos spacecraft. These two incidents alone created more than 5,000 larger than 10-centimeter space debris, which are still cluttering LEOs. The emergence of mega-constellations comprising thousands of LEO satellites and hundreds of MEO satellites accentuates concerns about collisions and conjunctions (or near-collisions) [5].

The large number of resident space objects (RSO) to be monitored requires that an automated sensor-based surveillance process be implemented, and be trusted. To achieve this, the risks of collision and of false alarms must be evaluated. In aviation navigation applications, these risks are measured in terms of integrity and continuity risks. Integrity is a measure of trust in sensor information [6]. Continuity is a system’s ability to operate without unscheduled interruptions [5]. These two safety performance metrics have requirements that are not application specific, and are codified for general system analysis in [7, 8]. For example, the destruction of an emergency communication satellite can be considered of major severity level, with an associated integrity risk requirement of $10^{-7}$ to $10^{-5}$ [8].

In risk evaluation, we want risk estimates to upper-bound the actual risk. Optimistic risk predictions are unacceptable because they can cause unsafe conditions. But, overly pessimistic predictions must be avoided, or the monitoring system may too often indicate that the automated system is not trustworthy. Therefore, the challenge is to provide tight risk bounds, starting with tight estimation error bounds on orbital elements or position coordinates used to predict space object motion.

This work focuses on two-line element (TLE) sets. As the de facto standard for distribution of RSO orbital elements, TLEs remain a vital initial estimate tool for space operations all over the world. Its limited accuracy and

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lack of covariance information have been the topic of many studies, but TLE error modeling is still mostly unexplored. Existing research [9–13] that deal with TLE accuracy often focus on TLE propagation errors using Simplified General Perturbation SGP4 propagation. This restricts the analysis to a few thousands of samples, and to a time period of just a few days up to a few months. There is no shortage of research on TLEs. However, with regard to risk assessment, these studies often encompass time scales too short to include a statistically significant number of samples. A different approach was taken in [14], where they estimated TLE uncertainties for several complete snapshots of the TLE catalogue for eight 60-day periods between 1990 and 2008. This work is promising in that it deduced a covariance look-up table for the entire SSN catalogue. However, the methodology compares states directly derived from TLEs with states resulting from orbit determination on pseudo-observations derived from TLE data. The problem is that the states derived from these pseudo-observations do not have known accuracy, and therefore do not provide a baseline for TLE errors.

In addition, TLE errors are unlikely to follow a known distribution. In this regard, overbounding theory can be exploited [1,2], as it currently is in aviation navigation applications [15–17]. First, overbounding provides the means to model empirical TLE data using familiar parametric distributions, Gaussian functions. Second, if errors for TLE orbital elements can be over-bounded, then orbit determination errors will be bounded as well, provided that the orbit estimator is linear or linearized. Under the assumptions described in [1,2], measurement error models propagated through linearized orbit determination processes provide bounding distributions on estimated spacecraft positions. These can then be used for collision risk evaluation. Unfortunately, there is currently no such overbounding error model for TLE data.

In response, in this paper, we collect, process, and analyze tens of thousands of TLE data samples for example MEO and GEO satellites. The TLE error data collection and processing method is described first in Section II of this paper. We then use Gaussian overbounding to establish a high-integrity, compact, parametric model of TLE data for MEO satellites in Section III, and for GEO satellites in Section IV. In future work, overbounding will help establish raw sensor observation models (including for telescopes and radars) in order to evaluate orbit determination and collision risk monitoring performance.

II. Method for Obtaining and Processing TLE Sets and ‘Truth’ Data

Figure 1 is the top level overview of the methodology described in this paper. We use online repositories from Space-track [18], the Crustal Dynamics Data Information System (CDDIS) [19], and the National Transportation Safety Board (NSTB) [20], which respectively contain TLE sets, GPS broadcast messages, and WAAS navigation messages. As will be explained shortly, CDDIS and NSTB information can be considered ‘truth’. We implemented the following process. First, TLE data is converted to position and velocity in Earth-Centered-Earth-Fixed (ECEF) coordinates. Then, truth data most closely matched to a reported TLE epoch is retrieved, propagated to that TLE epoch, and expressed in ECEF position and velocity. The TLE-to-truth difference gives the three-dimensional position and velocity error at the TLE epoch. This ECEF position and velocity error is then converted to radial, along-track, and cross-track positioning error. Overbounding functions for the resulting error distributions are determined using overbounding theory, and parameters of a high-integrity TLE error model are determined. The next sections describe the selection of candidate satellites, data acquisition and processing, and overbounding theory.

![Fig. 1](image-url) Overview of methodology for deriving the TLE error model developed in this paper
A. Selection of Candidate Satellites

In order to evaluate TLE errors, we consider space objects at known locations. Both Global Positioning System (GPS) and Wide Area Augmentation System (WAAS) satellites have a well maintained history of ephemerides with known accuracy \[21-23\] orders of magnitude higher than TLEs currently documented in the literature \[11-14\]. GPS and WAAS satellites were also chosen because they are observable from Tucson at any given time. This will facilitate the development of error models for actual telescope observations in future work.

Table 1  List of satellites studied in the paper

<table>
<thead>
<tr>
<th>Orbit Class</th>
<th>SATCAT no.</th>
<th>Satellite Name</th>
<th>PRN Signal</th>
<th>Launch Date</th>
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<tr>
<td>MEO</td>
<td>24876</td>
<td>USA-132</td>
<td>13</td>
<td>23 July 1997</td>
</tr>
<tr>
<td>MEO</td>
<td>25933</td>
<td>USA-145</td>
<td>11</td>
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<tr>
<td>MEO</td>
<td>26360</td>
<td>USA-150</td>
<td>20</td>
<td>11 May 2000</td>
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<td>MEO</td>
<td>26407</td>
<td>USA-151</td>
<td>28</td>
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<td>MEO</td>
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<td>USA-154</td>
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<td>MEO</td>
<td>27663</td>
<td>USA-166</td>
<td>16</td>
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<td>27704</td>
<td>USA-168</td>
<td>21</td>
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<td>MEO</td>
<td>28129</td>
<td>USA-175</td>
<td>22</td>
<td>21 December 2003</td>
</tr>
<tr>
<td>MEO</td>
<td>28190</td>
<td>USA-177</td>
<td>19</td>
<td>20 March 2004</td>
</tr>
<tr>
<td>MEO</td>
<td>28361</td>
<td>USA-178</td>
<td>23</td>
<td>23 June 2004</td>
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<td>28474</td>
<td>USA-180</td>
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<td>MEO</td>
<td>28874</td>
<td>USA-183</td>
<td>17</td>
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<td>USA-190</td>
<td>31</td>
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<td>MEO</td>
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<td>MEO</td>
<td>41328</td>
<td>USA-266</td>
<td>32</td>
<td>5 February 2016</td>
</tr>
<tr>
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<td>41589</td>
<td>Eutelsat 117 West B</td>
<td>131</td>
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<tr>
<td>GEO</td>
<td>33278</td>
<td>Inmarsat-4 F3</td>
<td>133</td>
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<td>GEO</td>
<td>28884</td>
<td>Galaxy 15</td>
<td>135</td>
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<tr>
<td>GEO</td>
<td>28868</td>
<td>Anik F1R</td>
<td>138</td>
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Each GPS satellite broadcasts a unique pseudo-random noise (PRN) signal numbered from 1 to 32. Thirty of the thirty-two currently operational GPS satellites are studied in this work, and heretofore identified by their assigned PRN. PRNs 4 and 18 were excluded for convenience in data processing, (because two different satellites were used to broadcast PRN 18 throughout the time period of study). There are nominally three operational WAAS satellites which also broadcast a unique PRN signal with identification number ranging from 131 to 138. Within the time period of our study, Inmarsat-4 F3 (assigned PRN 133), was retired and replaced with Eutelsat 117 West B (PRN 131). All four WAAS satellites were included in the study. The MEO and GEO satellites studied in this paper are listed in Table 1.

B. Acquiring TLE and Truth Data from Online Repositories

There are three types of data retrieved from three publicly accessible online repositories in this paper: (1) TLE sets stored by Space-Track, (2) GPS broadcast ephemerides stored by NASA’s CDDIS, and (3) WAAS navigation messages stored by the Federal Aviation Authority’s (FAA) NSTB. The TLEs were retrieved for each of the listed satellites for the time range of January 1, 1997 to March 31, 2018 for GPS, and January 1, 2014 to June 31, 2018 for WAAS.

Two-line element sets for objects tracked by the US military are made available to the general public by Air Force Space Command, through their website “space-track.org”. A TLE includes ten values, six of which represent parameters that describe an orbit (mean motion, mean anomaly, eccentricity, inclination, right ascension of the ascending node, argument of perigee). The remaining parameters (mean motion rate, mean motion acceleration, and a drag-like parameter) describe the effect of perturbations on the motion of the satellite \[24\]. The mathematical formulation of a TLE requires that periodic variations be removed to obtain the mean elements listed as parameters. To accurately retrieve a position and velocity vector requires that these periodic variations be reconstructed using a specific perturbation model. For satellite orbits with periods greater than 225 minutes, the Simplified Deep Perturbation 4 (SDP4) propagator is used to reconstruct the position and velocity vector. Each TLE data entry is valid only at the epoch listed, and errors grow significantly if propagated to a different time. ‘Truth’ data from CDDIS and NSTB is therefore propagated to the TLE epoch. CDDIS and NSTB data are considered ‘truth’ because they provide meter-level satellite positioning accuracy \[21\]–\[23\], whereas TLE errors are expected to exceed hundreds of meters \[11\]–\[14\].

The GPS broadcast ephemeris files contain navigation data broadcast by GPS satellites at regular two-hour intervals, and are valid over overlapping four-hour periods. Navigation files from multiple sites are merged into one coherent daily broadcast ephemeris file posted on the CDDIS website. A single satellite navigation message includes 17 parameters that describe the orbit (semi-major axis, mean motion, longitude of ascending node, inclination, and the rates of all four, mean anomaly, eccentricity, argument of perigee, and six harmonic parameters). A 17-parameter model given in \[25\] is used to propagate the spacecraft orbit to the reported TLE epoch.

The GEO navigation message broadcast by WAAS satellites contains the geostationary satellite’s orbit information. This enables use of the GEO satellite as an additional GPS-like ranging source. The message includes position, velocity, and acceleration of the satellite in Earth-Centered-Earth-Fixed (ECEF) coordinates, epoch time, and clock correction and health parameters. This ephemeris message is updated and rebroadcast every 60 seconds, and is valid for two-minute intervals. The WAAS broadcast satellite position is propagated to the TLE epoch using procedures presented in \[17\] and \[26\].

Time is expressed in a different reference frame between TLEs and the GPS and WAAS broadcast messages. TLE epochs are expressed in Julian date, while both GPS and WAAS messages are reported in GPS time (expressed in GPS weeks, and GPS seconds). GPS time leap seconds must be included for proper conversion.

C. Data Processing

Comparison of TLE sets with precise spacecraft positions are performed at the TLE epoch time to avoid introducing additional TLE propagation errors \[12\]. TLE data is converted to Earth-Centered-Earth-Fixed (ECEF) position and velocity for the same epoch time reported on the TLE using the SDP4 or the Simplified General Perturbations 4 (SGP4) algorithm. (In this work, using SGP4 instead of SDP4 gives similar results). Precise ephemeris timestamped closest to the reported TLE epoch are downloaded from CDDIS and NSTB for comparison. These precise ephemerides are propagated towards the TLE epoch, and the difference is computed. The error is expressed in a satellite-based reference frame, specifically the radial, along-track, and cross-track reference frame.
D. Overbounding Theory

*Overbounding theory* is a building block of integrity evaluation ([1] [2]). It leverages the fact that for safety, risk estimates do not need to be exact, but must upper-bound the actual risk. Thus, a finite set of measurements is sufficient to characterize a sensor (in this case the TLE database) with quantifiable integrity. The actual implementation of overbounding theory on error data is tackled in the discussion of Section III.B.

III. Statistical Analysis and Error Model for GPS MEO TLEs

A. TLE Error Analysis for GPS MEO Satellites

TLE errors for thirty GPS satellites are plotted in Figure 2. TLE data availability for GPS satellites over years 1998 to 2018 is described in Appendix, and explains the sparsity of data in the early years the study period. There were 146,660 total TLE error data points. Magnitudes of along-track errors are significantly larger than errors in the other two directions. An apparent shrinkage of error spread occurs sometime after July 2012, and again after the January 2015 mark. The cause for these are unknown; inquiries were sent to the Joint Space Operations Center (JSpOC).

Fig. 2 Error between TLEs and GPS broadcast ephemerides for PRNs 1-3, 5-17, and 19-32, from July 23, 1997 to March 31, 2018, expressed in radial, along-track, and cross-track reference frame

The top plot on Fig. 3 zooms in on the time when the TLE error behavior suddenly changes to form a more consistent pattern. There is more consistent behavior over recent years, with most TLE along-track errors ranging from -15 km to 15 km between March 2013 and October 2015, but staying within -6 to 6 km from October 2015 to 2018. Error data after October 21, 2015 is interpreted as stationary for statistical error analysis. The bottom plot in Fig. 3 shows that the time of this sudden change for GPS TLE error coincides with that for WAAS TLE error, which are further discussed in Section IV.
TLE errors sometimes exceed the 25,000 meters y-axis scale used in Fig. 2. Most of the ‘outlier’ data points could easily be identified, for example considering the consistency of TLE orbit parameters with previous TLEs. A detailed TLE error outlier analysis will be performed in future work. In this study, a 20,000 meters threshold is applied. Error values greater than the 20,000-meter hard limit were considered outliers. The number of outliers per year are plotted as a percentage of total error data points in Fig. 4. The percentage of outliers consistently decrease starting in year 2000. It was found that the number of outliers in 2015 drops from 74 to just 14 by 2017.
After outlier exclusion, TLE error sample statistics over the study period are given in Fig. 5 to reinforce our previous observations on the post-2015 stationary period. The graphs in Figure 5 show the yearly sample standard deviation and yearly sample mean of TLE errors over the entire study period. Yearly radial error standard deviations ranging from 1 km to 4 km from 1998 to 2015, but stays below approximately 0.5 km from 2016 to 2018. The standard deviation and mean are nearly constant from October 21, 2015 to 2018. These plots support the previously determined period where the statistics of the error are mostly stationary. The analysis in Section III.B is focused on the period which starts on 21 October 2015, 23:18:33 UTC, and ends on 1 April 2018, 00:00:00 UTC.

B. Gaussian Overbounding Model of TLE Errors for GPS MEO satellites

Gaussian overbounding functions are best represented on quantile-to-quantile (q-q) plots. A q-q plot is used to determine if a set of data comes from a known distribution. As depicted in Fig. 6, the q-q plot graphs the quantiles of TLE error data (y-axis) against the theoretical quantiles from a standard normal distribution (x-axis). If the distribution of TLE errors was normal, then the sample cumulative distribution curve would appear as a straight line, with slope the sample standard deviation and y-intercept the sample mean. But, TLE errors are not normally distributed.

On a q-q plot, a pair of overbounding functions can be found visually by sandwiching the sample error distribution between two curves, which can be straight lines if we consider Gaussian functions. This pair of Gaussian functions provide a conservative probabilistic model that can be used in risk analysis. In Fig. 6 the dashed lines that sandwich the radial, along-track, and cross-track error distributions are the overbounding lines. The slope and y-intercept of these overbounding lines represent the overbounding standard deviation, $\sigma_{ob}$, and bounding biases, $\pm \mu_{ob}$ respectively. The positive bias corresponds to the left overbound (above the actual error curve), and the negative bias to the right overbound. The legend shows the values for these slopes and biases, as well as the sample mean, $\mu$, and standard
deviation, $\sigma_s$. Using $\mu_s$ and $\sigma_s$ for the TLE error model in estimation would be optimistic because they only take into account the core of the distribution. For risk analysis to meet stringent requirements, $\sigma_{ob}$, and $\pm \mu_{ob}$ should be used instead.

Two example overbounding models are presented in Figures 6 and 7. The one in Fig. 6 covers the entire sample distribution made of more than $10^4$ data samples. If the process is stationary, this model will be valid approximately 99.99% of the time ($1 - 10^{-4}$). In parallel, in Fig. 7 we derived an overbounding model that only covers the core of the sample distribution, between $\pm 3.3$ on the x-axis, i.e., between $\pm 3.3$ standard deviations of the standard normal distribution, which corresponds to a $10^{-3}$ quantile. In Fig. 7 the values of $\sigma_{ob}$, and $\pm \mu_{ob}$ are much lower than in Fig. 6 but the model is only valid 99.9% of the time.

Fig. 6  Quantile-to-quantile plots of 99.99% paired overbounding model for 47,626 GPS MEO TLE error data points from October 21, 2015 to March 31, 2018

Fig. 7  Quantile-to-quantile plots of 99.9% paired overbounding for 47,626 GPS MEO TLE error data points from October 21, 2015 to March 31, 2018
IV. Statistical Analysis and Error Model for WAAS GEO TLEs

A. TLE Error Analysis for GEO WAAS Satellites

TLE errors for WAAS satellites are depicted in Figure 8. There were 5,813 total TLE error data points. Error magnitudes for GEOs are greater than MEO GPS TLE errors. Unlike Figure 2, along-track errors here are not as noticeably larger than in the other directions. For this GEO data set, cross-track errors are of the same magnitude as along-track errors. Similar to GPS TLE errors, the behavior of errors distinctly changes in year 2015. Figure 3, displayed on page 6 shows this parallel.

![Fig. 8](image)

**Fig. 8** Error between TLEs and NSTB navigation message for PRNS 131, 133, 135, and 138, from January 1, 2014 to June 30, 2018, expressed in radial, along-track, and cross-track reference frame

For TLE errors statistical analysis on WAAS GEO satellites, we exclude errors larger than 40,000 meters. Similar to the preceding MEO TLE error analysis, 40,000 meters is an arbitrary cut-off (of twice the value used for GPS TLE errors). The yearly sample standard deviations and means of GEO TLE errors are plotted in Fig. 9. Unlike MEO TLE errors, these statistics are not consistent, and the values for the standard deviation are close to twice the values we see for GPS in Fig. 5.

![Fig. 9](image)

**Fig. 9** Standard deviation and mean of TLE errors for WAAS GEO satellites
The percentage of outliers for the WAAS TLE data set is plotted in Fig. 10. Notice the difference in y-axis range for this figure and for Fig. 4. The percentage of outliers in GEO TLE errors is much lower than for MEO TLEs.

![Outlier statistics for WAAS GEO TLE errors](image)

**Fig. 10**  Outlier statistics for WAAS GEO TLE errors

### B. Gaussian Overbounding Model of TLE Errors for GEO WAAS Satellites

Figure 11 depicts the overbounding functions for WAAS TLE errors. Sample error distributions for the radial, along-track, and cross-track directions are all well-behaved without too many stray points on the tails of the distribution. This differs significantly from what was observed for GPS in Fig. 6, but the number of samples (3,979) is much lower than for GPS (47,626). The overbounding model that covers these more than $10^3$ data points will be deemed valid approximately $99.9\%$ of the time $(1 - 10^{-3})$. Sample standard deviations and means, as well as bounding standard deviations and bounding biases are given in legend in Fig. 11.

![Quantile-to-quantile plots of 99.9\% paired overbounding for 3,979 WAAS GEO TLE error points from October 21, 2015 to June 30, 2018](image)

**Fig. 11**  Quantile-to-quantile plots of 99.9\% paired overbounding for 3,979 WAAS GEO TLE error points from October 21, 2015 to June 30, 2018
V. Conclusion

In this paper, we quantified spacecraft positioning errors for 148,984 medium earth orbit (MEO) two-line element (TLE) sets and 5813 geostationary earth orbit (GEO) TLEs. The analysis shows that MEO TLE errors are significantly larger in the spacecraft along-track direction than they are in the cross-track and radial directions. The same is not true for GEO TLE errors, where along-track and cross-track errors are of similar magnitude. Also, TLE errors have significantly decreased over the past 20 years. In particular, after October 21, 2015, MEO GPS TLE errors become significantly smaller and more consistent as compared to their past history, whereas GEO WAAS TLE along-track and cross track errors tend to increase. High-integrity GPS MEO TLE error models were determined. For example, GPS TLE radial errors, which are not normally distributed, can be conservatively modeled using a pair of overbounding Gaussian functions with standard deviation 400 meters and bounding biases ±1200 meters. TLE errors for WAAS GEO satellites were about twice the magnitude of the errors for GPS MEO TLE sets. In future work, raw telescope data will be analyzed, and high-integrity telescope error models will be derived for orbit determination performance prediction.
Appendix

The following figure depicts the period of operation for each satellite studied in this paper. It is important to keep this in mind when interpreting plots presented in the body of the paper. The left starting point of each bar does not necessarily correspond to the first TLE available for that satellite. Instead, it corresponds to the time of the first broadcast message (‘truth’ data).

![Period of Operation for Investigated Satellites](image)

**Fig. 12** Period of operation for GPS and WAAS satellites investigated in this paper

References


[3] ESA's Space Debris Office at ESOC. “Space Debris by the Numbers.” Darmstadt, Germany. available online at https://www.esa.int/Our_Activities/Operations/Space_Debris/Space_debris_by_the_numbers


[18] Joint Space Operations Center. available online at www.space-track.org


